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VELOCITY OF FULL FLUIDIZATION OF A BED OF POLYDISPERSE GRANULAR MATERIALS

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Based on the principle of hydrodynamic equivalence, a technique to calculate the velocity of complete fluidization of polydisperse granular materials of different densities with allowance for the transformation of their initial size distribution has been worked out.

Keywords: *polydisperse fluidization bed, minimum of fluidization velocity, complete fluidization velocity, discrete size distribution of particles, continuous size distribution of particles, equivalent diameter of particles.*

Introduction. As is known [1], fluidization of polydisperse beds of granular materials differs substantially from fluidization of monodisperse particles. While the latter are fluidized immediately at a velocity u_{mf}, the fluidization of a polydisperse bed occurs gradually in a certain range of change of the velocity of filtration: $u_{min} - u_{f.f}$. At $u = u_{min}$ the finest fractions are fluidized that form a fluidized bed in the upper portion of a disperse packing. Closer to the gas-distributing grating there is a stationary blown-through bed of larger fractions. With increase in the gas velocity, the boundary between the fluidized and the fixed beds descends and at a certain velocity of filtration ($u_{f.f}$) this boundary reaches the gas distributor, and all of the particles get fluidized. Such a velocity is called the velocity of full fluidization and it plays the part of an analog of the minimum velocity of fluidization for a bed of monodisperse particles.

Minimum Fluidization Velocity. The procedure of determining the minimum fluidization velocity has been well worked out both experimentally and theoretically. There are numerous recommendations for its calculation. The most well-known is the Todes formula [2]:

$$\mathrm{Re}_{mf} = \frac{u_{mf} d}{\nu_f} = \frac{\mathrm{Ar}}{1400 + 5.22\sqrt{\mathrm{Ar}}} , \quad (1)$$

which gives a good description of the influence exerted on u_{mf} not only by the diameter of particles but also by the pressure and temperature of the fluidizing gas.

Full Fluidization Velocity. As our analysis of the phenomenon shows, it is more efficient to construct the procedure of calculation of the full fluidization velocity on the basis of Eq. (1). Here, the main problem is the determination of the effective (equivalent) diameter of a mixture of particles d_{eq} allowing one to use Eq. (1) for calculating $u_{f.f}$, having replaced d by d_{eq}.

We will consider two important particular cases.

1. The particles have the same density. In [3], on the basis of experimental investigations of beds of agglomerate of wide fractional composition, a simple calculating formula was derived to calculate d_{eq}:

$$d_{eq} = \langle d \rangle = \sum_{i=1}^{N} d_i \eta_i . \quad (2)$$

Thus, d_{eq} represents the arithmetic mean of the diameters of fractions d_i. The velocity of full fluidization, with allowance for (2), is determined from the relation that follows from Eq. (1):

$$\mathrm{Re}_{f.f} = \frac{u_{f.f} d_{eq}}{\nu_f} = \frac{\mathrm{Ar}_{eq}}{1400 + 5.22\sqrt{\mathrm{Ar}_{eq}}} . \quad (3)$$

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2. Particles have different densities. This case is of interest in calculation of fluidized-bed boilers, when mixtures of various fuels are burnt simultaneously [4]. Because of the difference in the densities of the inert substance and the types of fuel burnt, the direct use of Eq. (2) presents difficulties, and a special technique is required for determining d_{eq} that accounts for this difference.

Discrete Distribution of Fractions. Since in fluidized-bed furnaces the particles of an inert substance constitute the main fraction of dispersed material, we will consider separately different types of fuel and of inert substance, representing them by the following density and size distributions:

$$\rho_s^{(k)}, d_i^{(k)}, \eta_i^{(k)}, \quad k = 1, ..., n, \quad i = 1, ..., N_k; \tag{4}$$

$$\rho_{in}, d_{in}^{(i)}, \eta_{in}^{(i)}, \quad i = 1, ..., N_{in}. \tag{5}$$

For concentrations $\eta_i^{(k)}$ and $\eta_{in}^{(k)}$ the following equalities are valid:

$$\sum_{i=1}^{N_k} \eta_i^{(k)} = 1, \quad k = 1, ..., n; \quad \sum_{i=1}^{N_{in}} \eta_{in}^{(i)} = 1. \tag{6}$$

In order to use Eq. (2) for a mixture of different materials, the fractional composition of which is given by distributions (4) and (5), it is obviously necessary to bring them to a unified density of particles. As such, it is reasonable to select the density of the inert substance ρ_{in} that constitutes the main portion of the mass of the bed. The transformation of the distributions was made on the basis of the principle of hydrodynamic equivalence: Two spherical particles with the characteristics d_1, ρ_1 and d_2, ρ_2 are hydrodynamically equivalent if their minimum velocities of fluidization are equal. Assuming that $\rho_1 = \rho_{in}$ and $\rho_2 = \rho_s^{(k)}$, the distribution (4) can be transformed as

$$\rho_{in}, d_{in}^{(i,k)}, \eta_i^{(k)}, \quad k = 1, ..., n, \quad i = 1, ..., N_k, \tag{7}$$

where $d_{in}^{(i,k)}$ is the diameter of the particles of the ith fraction of the kth type of fuel reduced to the density of the inert substance. The concrete realization of the principle of hydrodynamic equivalence is based on the use of the Todes formula (1). For the above-indicated particles with the characteristics d_1, ρ_1 and d_2, ρ_2 we can write down the condition of the equality of their velocities of minimum fluidization:

$$\frac{\nu_f}{d_1} \frac{Ar_1}{1400 + 5.22\sqrt{Ar_1}} = \frac{\nu_f}{d_2} \frac{Ar_2}{1400 + 5.22\sqrt{Ar_2}}. \tag{8}$$

This relation can be considered as a transcendental equation for finding the value of d_1/d_2. For its solution we will consider two limiting cases: fine particles, $5.22\sqrt{Ar} << 1400$, for which solution (8) has the form

$$\frac{d_1}{d_2} = \sqrt{\frac{\rho_2}{\rho_1}}; \tag{9}$$

large particles, $5.22\sqrt{Ar} >> 1400$, for which Eq. (8) yields

$$\frac{d_1}{d_2} = \frac{\rho_2}{\rho_1}. \tag{10}$$

In the general case, for d_1/d_2 from Eq. (8) it follows that

$$\frac{d_1}{d_2} = \sqrt{\frac{\rho_2}{\rho_1} \frac{1400 + 5.22\sqrt{Ar_1}}{1400 + 5.22\sqrt{Ar_2}}}. \tag{11}$$

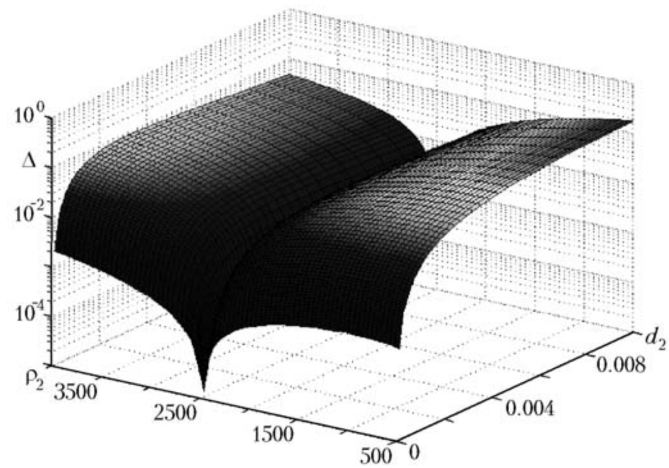

Fig. 1. Dependence of the error of Eq. (12) relative to exact solution (11) on the density and size of particles. $\rho_1 = 2500$ kg/m^3. d_2, m; ρ_2, kg/m^3.

Substituting $d_1 = (\rho_2/\rho_1)d_2$ into Ar_1 from (10), we obtain an approximate solution of (11) in the form

$$\frac{d_1}{d_2} = \sqrt{\frac{\rho_2}{\rho_1}\frac{1400 + 5.22\dfrac{\rho_2}{\rho_1}\sqrt{\mathrm{Ar}_2}}{1400 + 5.22\sqrt{\mathrm{Ar}_2}}}\ . \tag{12}$$

Figure 1 compares the exact numerical solution of Eq. (11) and calculation by Eq. (12). As is seen, the maximum error of the approximate solution (12) does not exceed 11% and is attained for large and light particles. Equation (12) allows us to calculate the transition from particles with the diameter d_2 and density ρ_2 to an equivalent particle of diameter $d_1 = d_2\sqrt{\frac{\rho_2}{\rho_1}\frac{1400 + 5.22\frac{\rho_2}{\rho_1}\sqrt{\mathrm{Ar}_2}}{1400 + 5.22\sqrt{\mathrm{Ar}_2}}}$ having the density ρ_1. For $\rho_1 = \rho_{\mathrm{in}}$ and $\rho_2 = \rho_{\mathrm{s}}^{(k)}$, $d_2 = d_i^{(k)}$ from Eq. (12) we obtain an equation for calculating the quantities $d_1 = d_{\mathrm{in}}^{(i,k)}$ that enter into the sought distribution (7):

$$d_{\mathrm{in}}^{(i,k)} = d_i^{(k)}\sqrt{\frac{\rho_{\mathrm{s}}^{(k)}}{\rho_{\mathrm{in}}}\frac{1400 + 5.22\dfrac{\rho_{\mathrm{s}}^{(k)}}{\rho_{\mathrm{in}}}\sqrt{\mathrm{Ar}_i^{(k)}}}{1400 + 5.22\sqrt{\mathrm{Ar}_i^{(k)}}}}\ . \tag{13}$$

Figure 2 presents the ratios $d_{\mathrm{in}}^{(i,k)}/d_i^{(k)}$ for different $d_i^{(k)}$, T, and p.

The equivalent diameter of the particles of the bed after all the particles were reduced to a unified density can be calculated from Eq. (2) which, subject to distributions (5) and (7), has the form

$$d_{\mathrm{eq}} = \eta_{\mathrm{in}}\sum_{i=1}^{N_{\mathrm{in}}} d_{\mathrm{in}}^{(i)}\eta_{\mathrm{in}}^{(i)} + (1 - \eta_{\mathrm{in}})\sum_{k=1}^{n}\sum_{i=1}^{N_k} d_{\mathrm{in}}^{(i,k)}\eta_i^{(k)}\eta^{(k)}\ . \tag{14}$$

Here the following condition is valid:

$$\sum_{k=1}^{n}\sum_{i=1}^{N_k}\eta_i^{(k)}\eta^{(k)} = 1\ . \tag{15}$$

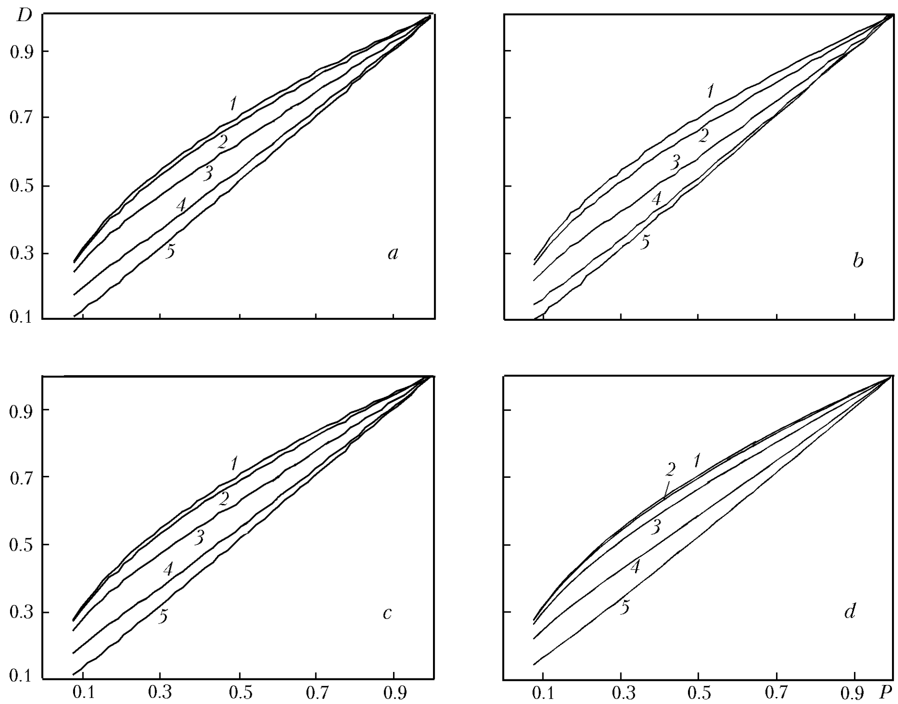

Fig. 2. Dependence of $D = d_{\text{in}}^{(i,k)}/d_i^{(k)}$ on $P = \rho_s^{(k)}/\rho_{\text{in}}$ at T = 300 K (a, b) and 1300 K (c, d), p = 0.1 MPa (a, c) and 0.6 MPa (b, d): 1) $d_i^{(k)}$ = 0.1 mm; 2) 0.32; 3) 1; 4) 3.16; 5) 10.

The velocity of full fluidization of the presented mixture of particles is calculated by the formula

$$\text{Re}_{\text{f.f}} = \frac{u_{\text{f.f}} d_{\text{eq}}}{\nu_{\text{f}}} = \frac{\text{Ar}_{\text{eq}}^*}{1400 + 5.22\sqrt{\text{Ar}_{\text{eq}}^*}} . \tag{16}$$

It should be noted that in calculating $u_{\text{f.f}}$ with the use of d_{eq} defined by Eq. (14) it was assumed that at $u = u_{\text{f.f}}$ the particles of all the fractions remain in a fluidized bed. Actually the bed cannot contain particles whose diameter is smaller than the levitation diameter. These particles will be entrained from the bed, the initial size distribution in the supplying flow will be changed and, correspondingly, the values of d_{eq} and $u_{\text{f.f}}$ will also change. It is clear that the actual values of d_{eq} and $u_{\text{f.f}}$ are calculated in the iterative cycle (see below).

Continuous Distribution of Fractions. The above results can be easily generalized to the case of continuous distribution of fractions. In this case Eq. (14) will have the form

$$d_{\text{eq}} = \eta_{\text{in}} \left\langle d_{\text{in}} \right\rangle + (1 - \eta_{\text{in}}) \sum_{k=1}^{n} \left\langle d_{\text{in}}^{(k)} \right\rangle \eta^{(k)} , \tag{17}$$

where

$$\left\langle d_{\text{in}} \right\rangle = \int_{(d_{\text{in}})_{\min}}^{(d_{\text{in}})_{\max}} d_{\text{in}} f(d_{\text{in}})\, d\,(d_{\text{in}}) , \quad \left\langle d_{\text{in}}^{(k)} \right\rangle = \int_{(d_{\text{in}}^{(k)})_{\min}}^{(d_{\text{in}}^{(k)})_{\max}} d_{\text{in}}^{(k)} \varphi\left(d_{\text{in}}^{(k)}\right) d\left(d_{\text{in}}^{(k)}\right) .$$

We note that the quantities $(d_{\text{in}})_{\min}$ and $(d_{\text{in}}^{(k)})_{\min}$ should be at least not smaller than the levitation diameter of the particle of density ρ_{in}. This quantity is calculated by the formula [5]

$$d_t = 0.18 \frac{u_{f.f}^2}{g} \frac{\rho_f}{\rho_{in} - \rho_f} \left(1 + \sqrt{1 + 556 \frac{g\nu_f}{u_{f.f}^3} \frac{\rho_{in} - \rho_f}{\rho_f}} \right). \quad (18)$$

Since, generally speaking, the initial distribution (5), (7) may contain particles of diameter smaller than d_t, the values $(d_{in})_{min}$, $(d_{in}^{(k)})_{min}$, d_{eq}, $u_{f.f}$ are determined from the results of realization of the iteration process similar to that described in [6]:

1) from the given fractional composition of the supplied fuel and inert substance the values of $(d_{eq})_0$ and $(u_{f.f})_0$ are determined within the framework of the developed technique;

2) the levitation diameter d_{t0} is calculated by Eq. (18) for $(u_{f.f})_0$;

3) all the particles whose diameters are smaller than d_{t0} are excluded from the distributions of d_{in} and $d_{in}^{(k)}$, and the values of $(d_{in})_{min1}$ and $(d_{in}^{(k)})_{min1}$ are determined;

4) the values of $(d_{eq})_1$ and $(u_{f.f})_1$ are found using the new fractional composition of particles;

5) the value of d_{t1} is calculated by Eq. (18) for $(u_{f.f})_1$.

The process terminates when the condition $|1 - (u_{f.f})_{k-1}/(u_{f.f})_k| < \varepsilon$ is satisfied (k is the number of the iteration cycle). The quantity $(u_{f.f})_k$ is considered to be the velocity of full fluidization of a polydisperse mixture of particles. The above is also equally valid for the case of discrete distribution of fractions.

Conclusions. Using the principle of hydrodynamic equivalence, the technique of calculation of the velocities of full fluidization of polydisperse different-density particles has been developed including the calculation of the equivalent diameter of a mixture (14), (17). The iteration cycle of the calculation of the value of $u_{f.f}$ is presented.

NOTATION

$\mathrm{Ar} = \frac{gd^3}{\nu_f^2}\left(\frac{\rho_s}{\rho_f} - 1\right)$, $\mathrm{Ar}_1 = \frac{gd_1^3}{\nu_f^2}\left(\frac{\rho_1}{\rho_f} - 1\right)$, $\mathrm{Ar}_2 = \frac{gd_2^3}{\nu_f^2}\left(\frac{\rho_2}{\rho_f} - 1\right)$, $\mathrm{Ar}_{eq} = \frac{gd_{eq}^3}{\nu_f^2}\left(\frac{\rho_s}{\rho_f} - 1\right)$, $\mathrm{Ar}_{eq}^* = \frac{gd_{eq}^3}{\nu_f^2}\left(\frac{\rho_{in}}{\rho_f} - 1\right)$, $\mathrm{Ar}_i^{(k)} = \frac{g(d_i^{(k)})^3}{\nu_f^2}\left(\frac{\rho_s^k}{\rho_f} - 1\right)$, Archimedean numbers; d, particle diameter, m; $f(d_{in})$, function of distribution of the values of d_{in}; g, free fall acceleration, m/sec^2; n, number of types of fuel; N, number of fractions; Re, Reynolds number; u, filtration velocity, m/sec; ε, Δ, relative errors of calculation of $u_{f.f}$ and d_1/d_2, respectively; η, mass fraction (concentration); ν_f, kinematic viscosity of a gas, m^2/sec; ρ, density, kg/m^3; $\varphi(d_{in}^{(k)})$, function of distribution of the values of $d_{in}^{(k)}$. Superscripts: i, number of a fraction; k, number of the type of fuel. Subscripts: f, gas; f.f, full fluidization; eq, equivalent; i, number of a fraction; in, inert substance; k, number of iteration cycle; mf, minimum fluidization; min, minimum; max, maximum; s, particles; t, conditions for levitation of a single particle of density ρ_{in}.

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